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# Lecture Notes in Earth Sciences

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## Numerical Geology

A Source Guide, Glossary and Selective Bibliography  
to Geological Uses of Computers and Statistics



Springer-Verlag

Berlin Heidelberg New York London Paris Tokyo



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